COBEM 99
XV Congresso Brasileiro de Engenharia Mecânica
15th Brazilian Congress of Mechanical Engineering
22 - 26 de Novembro de 1999 / November 22 - 26, 1999 Águas de Lindóia, São Paulo,

LAMINAR FORCED CONVECTION THROUGH RECTANGULAR DUCTS WITH UNIFORM AXIAL AND PERIPHERAL HEAT FLUX

Cláudio Lindquist
João Batista Aparecido
UNESP / Ilha Solteira, Departamento de Engenharia Mecânica
Av. Brasil Centro, 56 – 15385-000 – Ilha Solteira, SP, Brasil

Abstract. *Hydrodynamically developed, thermally developing laminar flow of a Newtonian fluid inside straight rectangular ducts with boundary uniform axial and peripheral heat flux is analytically studied using the generalized integral transform technique. Laminar fluid flow, constant fluid properties, high Péclet number and negligible viscous dissipation hypothesis are employed. The energy equation for the unknown temperature distribution is transformed by the use of the integral transform technique resulting in a coupled system of first order ordinary differential equations for the unknown transformed temperature profile. Such system is solved and the temperature profile can be retrieved by using the inversion formula. Numerical results are presented for quantities of practical interest within the thermal entry and the fully developed region, for a wide range of the axial distance and several aspect ratios. Some of those thermal quantities are: bulk fluid temperature, average wall temperature, average and local Nusselt numbers.*

Keywords: *Forced convection, Rectangular duct, Generalized integral transform technique.*

1. INTRODUCTION

The increasing needs for energy savings has motivated the development of lighter, more economic and efficient heat exchanger devices. These needs have greatly stimulated the research in the heat transfer characteristics for tubes of various cross sections. Heat transfer solutions for laminar forced convection through rectangular channels is of great interest, as these are often employed in several heat-exchange devices, like compact heat exchangers, solar collectors, nuclear reactor plate-type fuel assemblies, and several other devices.

Exact heat transfer solutions for laminar forced flow inside rectangular ducts are quite desirable for both reference purposes and validation of numerical and approximate schemes, especially for thermally developing flows. Except for ducts with simpler geometry, given by a single coordinate, the available analytical works found in the literature are quite scarce. The rectangular channel is a typical example, with the difficulties that arise from the solution of multidimensional convection problems, demanding costly numerical solutions, and limited to regions away from the inlet.

The extension of the generalized integral transform technique to solve three-dimensional, non-separable, convection-diffusion problems within irregular shaped domains was done by Aparecido & Cotta (1990a, 1992). Further, Aparecido & Cotta (1990b) applied such technique to solve thermally developing laminar flow inside rectangular ducts for boundary condition of the first kind.

In this paper, the hydrodynamically developed and thermally developing laminar flow of a Newtonian fluid inside a rectangular duct with boundary uniform axial and peripheral heat flux is analytically studied employing the generalized integral transform technique. Several quantities of practical interest are given within the thermal entry and the fully developed regions, for a wide range of the axial distance and several aspect ratios.

2. ANALYSIS

The present study deals with the laminar flow of a Newtonian fluid inside a rectangular duct, having a fully developed velocity profile and subjected to uniform axial and peripheral heat flux. The rectangular duct and its coordinate system are depicted in Figure 1.

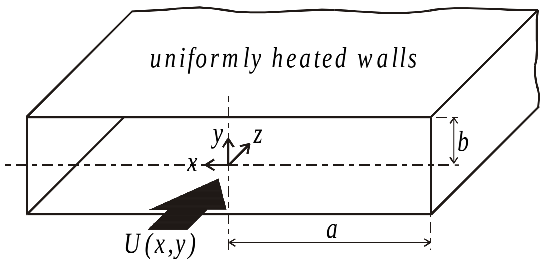

Figure 1. Coordinate system for the rectangular duct with axial and peripheral heat flux.

The energy equation for constant physical properties, when viscous dissipation may be neglected, is written as

$$\rho c_p u(x,y)\frac{\partial T(x,y,z)}{\partial z} = k\left[\frac{\partial^2 T(x,y,z)}{\partial x^2} + \frac{\partial^2 T(x,y,z)}{\partial y^2}\right], 0 < x < a,\ 0 < y < b,\ z > 0 \quad \text{(1a)}$$

with inlet and boundary conditions given by

$$T(x,y,0) = T_o,\quad 0 \le x \le a,\quad 0 \le y \le b \quad \text{(1b)}$$

$$\left.\frac{\partial T(x,y,z)}{\partial x}\right|_{x=0} = 0,\quad \left.\frac{\partial T(x,y,z)}{\partial x}\right|_{x=a} = \frac{q''}{k},\quad z > 0; \quad \text{(1c,d)}$$

$$\left.\frac{\partial T(x,y,z)}{\partial y}\right|_{y=0} = 0,\quad \left.\frac{\partial T(x,y,z)}{\partial y}\right|_{y=b} = \frac{q''}{k},\quad z > 0. \quad \text{(1e,f)}$$

The dimensionless form of Eqs. (1) may be written as

$$U(X,Y)\frac{\partial \Theta(X,Y,Z)}{\partial Z} = \frac{\partial^2 \Theta(X,Y,Z)}{\partial X^2} + \frac{\partial^2 \Theta(X,Y,Z)}{\partial Y^2}, 0 < X < \alpha, 0 < Y < \beta, Z > 0; \quad \text{(2a)}$$

$$\Theta(X,Y,0)=0, \quad 0\le X\le\alpha, \quad 0\le Y\le\beta\,; \tag{2b}$$

$$\left.\frac{\partial\Theta(X,Y,Z)}{\partial X}\right|_{X=0}=0, \quad \left.\frac{\partial\Theta(X,Y,Z)}{\partial X}\right|_{X=\alpha}=1, \quad Z>0\,; \tag{2c,d}$$

$$\left.\frac{\partial\Theta(X,Y,Z)}{\partial Y}\right|_{Y=0}=0, \quad \left.\frac{\partial\Theta(X,Y,Z)}{\partial Y}\right|_{Y=\beta}=1, \quad Z>0\,; \tag{2e,f}$$

where the following dimensionless groups were defined:

$$\begin{gathered} X=\frac{x}{D_h}, \qquad Y=\frac{y}{D_h}, \qquad Z=\frac{z}{D_h\,Pe}, \quad \alpha=\frac{a}{D_h}, \qquad \beta=\frac{b}{D_h}, \\ \Theta(X,Y,Z)=\frac{T(x,y,z)-T_o}{q''D_h/k}, \quad U(X,Y)=\frac{u(x,y)}{u_m}, \qquad Pe=\frac{\rho c_p}{k}u_m D_h\,; \end{gathered} \tag{3a}$$

with

$$D_h=\frac{4A_c}{P}=\frac{4ab}{a+b}. \tag{3b}$$

The dimensionless velocity profile is given by Aparecido & Cotta (1990b) as an infinite series in the following form

$$U(X,Y)=A^*\sum_{k=1,3,\ldots}^{\infty}E_kF_k(Y)G_k(X) \tag{4a}$$

where

$$A^*=\frac{48}{\pi^3\left[1-\dfrac{192}{\pi^5\alpha^*}\displaystyle\sum_{k=1,3,\ldots}^{\infty}\frac{\tanh(a_k\beta)}{k^5}\right]}, \quad E_k=\frac{(-1)^{(k-1)/2}}{k^3}, \qquad a_k=\frac{k\pi}{2\alpha}; \tag{4b,c,d}$$

$$F_k(Y)=1-\frac{\cosh(a_kY)}{\cosh(a_k\beta)}, \qquad G_k(X)=\cos(a_kX), \qquad \alpha^*=\frac{b}{a}. \tag{4e,f,g}$$

In order to set all the boundary conditions to homogeneous, we take the following variable transformation

$$\theta(X,Y,Z)=\Theta(X,Y,Z)-\frac{1}{2}\left(\frac{X^2}{\alpha}+\frac{Y^2}{\beta}\right) \tag{5}$$

Then, Eqs. (2) becomes

$$U(X,Y)\frac{\partial\theta(X,Y,Z)}{\partial Z}=\frac{\partial^2\theta(X,Y,Z)}{\partial X^2}+\frac{\partial^2\theta(X,Y,Z)}{\partial Y^2}+4\,, \; 0<X<\alpha,\; 0<Y<\beta,\; Z>0 \tag{6a}$$

with inlet and boundary conditions given by

$$\theta(X,Y,0) = -\frac{1}{2}\left(\frac{X^2}{\alpha} + \frac{Y^2}{\beta}\right), \quad 0 \le X \le \alpha, \quad 0 \le Y \le \beta\,; \tag{6b}$$

$$\left.\frac{\partial\theta(X,Y,Z)}{\partial X}\right|_{X=0} = 0, \quad \left.\frac{\partial\theta(X,Y,Z)}{\partial X}\right|_{X=\alpha} = 0, \quad Z > 0\,; \tag{6c,d}$$

$$\left.\frac{\partial\theta(X,Y,Z)}{\partial Y}\right|_{Y=0} = 0, \quad \left.\frac{\partial\theta(X,Y,Z)}{\partial Y}\right|_{Y=\beta} = 0, \quad Z > 0\,. \tag{6e,f}$$

3. SOLUTION

Due to the non-separable nature of problem defined by Eqs. (6), it cannot be solved through the classical Integral Transform Technique - ITT (Mikhailov & Özisik, 1984). This analytical method, though a powerful tool for solving linear boundary value problems found in heat transfer applications, cannot deal with problems involving non-separable eigenvalue systems. To overcome that and other limitations, several ideas have been presented in order to extend the classical ITT after the pioneering work of Özisik & Murray (1974). The so-called Generalized Integral Transform Technique (GITT) that is an extension of the Integral Transform Technique (ITT) can handle more general problems. For further information on the GITT, the reader may refer to the work of Cotta (1993). The use of the GITT can be summarized by the following basic steps.

Choosing and Solving the Eigenvalue Problems

The eigenvalue problems in both coordinate variables *X* and *Y* are chosen (Aparecido, 1997) as

$$\frac{d^2\psi(\mu,X)}{dX^2} + \mu^2\psi(\mu,X) = 0, \quad 0 < X < \alpha\,; \tag{7a}$$

$$\left.\frac{d\psi(\mu,X)}{dX}\right|_{X=0} = 0, \quad \left.\frac{d\psi(\mu,X)}{dX}\right|_{X=\alpha} = 0\,; \tag{7b,c}$$

and

$$\frac{d^2\phi(\lambda,Y)}{dY^2} + \lambda^2\phi(\lambda,Y) = 0, \quad 0 < Y < \beta\,; \tag{8a}$$

$$\left.\frac{d\phi(\lambda,Y)}{dY}\right|_{Y=0} = 0, \quad \left.\frac{d\phi(\lambda,Y)}{dY}\right|_{Y=\beta} = 0\,; \tag{8b,c}$$

which may be readily solved to yield the eigenfunctions and eigenvalues as

$$\psi_i(X) = B_i \cos(\mu_i X), \qquad \phi_m(Y) = C_m \cos(\lambda_m Y)\,; \tag{9a,b}$$

$$\mu_i = \frac{(i-1)\pi}{\alpha}, \qquad \lambda_m = \frac{(m-1)\pi}{\beta}. \qquad (9c,d)$$

The parameters B_i and C_m are nonzero arbitrary constants, and they are chosen so that the orthogonal functions $\psi_i(X)$ and $\phi_m(Y)$ be also orthonormal. Then

$$B_i = \left[\frac{\alpha}{2}\left(1+\delta_{i1}\right)\right]^{-1/2}, \qquad C_m = \left[\frac{\beta}{2}\left(1+\delta_{m1}\right)\right]^{-1/2}, \qquad i,m = 1,2,3,... \qquad (9e,f)$$

where δ_{ij} or δ_{mn} is the Kronecker delta.

Defining the Integral Transform Pair

The integral transform pair with respect to the ψ and ϕ variables is stated as

$$\textit{Transform:}\ \tilde{\tilde{\theta}}_{im}(Z) = \int_0^{\alpha}\int_0^{\beta} \psi_i(X)\phi_m(Y)\theta(X,Y,Z)dYdX\ ; \qquad (10a)$$

$$\textit{Inversion:}\quad \theta(X,Y,Z) = \sum_{i=1}^{\infty}\sum_{m=1}^{\infty} \psi_i(X)\phi_m(Y)\tilde{\tilde{\theta}}_{im}(Z)\,. \qquad (10b)$$

Transforming the Problem Formulation

Operating the integral transform on the energy equation with the transforms defined above, one can obtain the following infinite system of coupled ordinary differential equations for the transformed dimensionless temperature (Aparecido & Cotta, 1990b)

$$\sum_{j=1}^{\infty}\sum_{n=1}^{\infty} A_{ijmn}\frac{\partial\tilde{\tilde{\theta}}_{jn}(Z)}{\partial Z} + (\mu_i^2 + \lambda_m^2)\tilde{\tilde{\theta}}_{im}(Z) = \tilde{\tilde{S}}_{im}, \quad Z > 0 \quad (i,m = 1,2,3,...)\ ; \qquad (11a)$$

where

$$A_{ijmn} = \int_0^{\alpha}\int_0^{\beta} \psi_i(X)\psi_j(X)\phi_m(Y)\phi_n(Y)U(X,Y)dYdX\ ; \qquad (11b)$$

$$\tilde{\tilde{S}}_{im} = 4\delta_{i1}\delta_{m1}\sqrt{\alpha\beta}\,. \qquad (11c)$$

The transform of the inlet condition becomes

$$\tilde{\tilde{\theta}}_{im}(0) = \tilde{\tilde{g}}_{im} = \int_0^{\alpha}\int_0^{\beta} \psi_i(X)\phi_m(Y)\theta(X,Y,0)dYdX\ ; \qquad (12a)$$

so that

$$\tilde{\tilde{\theta}}_{11}(0) = -(\alpha+\beta)\frac{\sqrt{\alpha\beta}}{6}, \qquad \tilde{\tilde{\theta}}_{1m}(0) = \frac{(-1)^m}{\lambda_m^2}\sqrt{\frac{2\alpha}{\beta}} \quad (m = 2,3,4,...\infty)\ ; \qquad (12b,c)$$

$$\tilde{\tilde{\theta}}_{i1}(0) = \frac{(-1)^i}{\mu_i^2}\sqrt{\frac{2\beta}{\alpha}} \quad (i = 2,3,4,...\infty), \qquad \tilde{\tilde{\theta}}_{im}(0) = 0 \quad (i,m = 2,3,4,...\infty)\,. \qquad (12d,e)$$

The double integral in Eq. (11b) is evaluated to provide

$$A_{ijmn} = A^* \sum_{k=1,3,...}^{\infty} E_k \omega_{ijk} \xi_{mnk} \tag{13a}$$

where

$$\omega_{ijk} = \frac{(-1)^{\frac{k-1}{2}+i+j} B_i B_j}{4} \left(\frac{1}{\mu_i + \mu_j + a_k} - \frac{1}{\mu_i + \mu_j - a_k} + \frac{1}{\mu_i - \mu_j + a_k} - \frac{1}{\mu_i - \mu_j - a_k} \right)$$

and (13b,c)

$$\xi_{mnk} = \delta_{mn} + \frac{(-1)^{m+n+1} C_m C_n a_k \tanh(a_k \beta)}{2} \left[\frac{1}{(\lambda_m + \lambda_n)^2 + a_k^2} + \frac{1}{(\lambda_m - \lambda_n)^2 + a_k^2} \right].$$

Truncating the Infinite System

The system given by Eqs. (11) provides an infinite number of coupled first order ordinary differential equations for the transformed dimensionless temperature subjected to the transformed initial condition given by Eqs. (12). The analysis hitherto presented is evidently formal and exact, but in order to yield numerical results the infinite system (11) has to be truncated to a sufficiently large finite dimension as shown bellow

$$\sum_{j=1}^{N} \sum_{n=1}^{N} A_{ijmn} \frac{\partial \tilde{\tilde{\theta}}_{jn}(Z)}{\partial Z} + (\mu_i^2 + \lambda_m^2) \tilde{\tilde{\theta}}_{im}(Z) = \tilde{\tilde{S}}_{im} , \quad Z > 0 \quad (i,m = 1,2,3,..., N) \tag{14a}$$

subjected to the initial condition

$$\tilde{\tilde{\theta}}_{im}(0) = \tilde{\tilde{g}}_{im} \quad (i, m = 1,2,3,..., N), \tag{14b}$$

where N is the number of terms in the truncated series, chosen sufficiently large to provide the desired accuracy. The finite system (14) of N^2 coupled equations may be written in a matrix form as

$$\boldsymbol{A}\, \boldsymbol{y}'(Z) + \boldsymbol{B}\, \boldsymbol{y}(Z) = \boldsymbol{c}\,; \tag{15a}$$

$$\boldsymbol{y}(0) = \boldsymbol{g}\,; \tag{15b}$$

where the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$, and the vectors $\boldsymbol{c}$ and $\boldsymbol{g}$ are a proper representation of the scalar coefficients A_{ijmn}, $(\mu_i^2 + \lambda_m^2)$, $\tilde{\tilde{S}}_{im}$ and $\tilde{\tilde{g}}_{im}$, respectively (Aparecido & Cotta, 1990b). The vector $\boldsymbol{y}(Z)$ is a representation of the dimensionless temperature transform and is defined as

$$\begin{aligned} \boldsymbol{y}(Z) = [&\tilde{\tilde{\theta}}_{11}(Z), \tilde{\tilde{\theta}}_{12}(Z), ..., \tilde{\tilde{\theta}}_{1N}(Z), \tilde{\tilde{\theta}}_{21}(Z), \tilde{\tilde{\theta}}_{22}(Z), ..., \tilde{\tilde{\theta}}_{2N}(Z) \\ &..., \tilde{\tilde{\theta}}_{N1}(Z), \tilde{\tilde{\theta}}_{N2}(Z), ..., \tilde{\tilde{\theta}}_{NN}(Z)]^{\mathrm{T}}. \end{aligned} \tag{16}$$

Multiplying Eq. (15a) by the inverse of matrix $\boldsymbol{A}$, it becomes

$$\boldsymbol{y}'(Z) = \boldsymbol{D}\, \boldsymbol{y}(Z) + \boldsymbol{e} \tag{17a}$$

where $\boldsymbol{D} = -\boldsymbol{A}^{-1}\boldsymbol{B}$ and $\boldsymbol{e} = \boldsymbol{A}^{-1}\boldsymbol{c}$. (17b)

Solving the Finite System

The finite system (17) can be solved using packed subroutines for solving initial value problems, such as DIVPAG from the IMSL package (Visual Numerics, 1994), providing the system solution with high accuracy.

Once the system solution has been found, the inversion formula Eq. (10b) combined with Eqs. (5) and (16) is recalled to compute the dimensionless temperature profile

$$\Theta(X,Y,Z) = \sum_{i=1}^{N}\sum_{m=1}^{N} B_i C_m \cos(\mu_i X)\cos(\lambda_m Y)\tilde{\tilde{\theta}}_{im}(Z) + \frac{1}{2}\left(\frac{X^2}{\alpha} + \frac{Y^2}{\beta}\right). \tag{18}$$

Computing the Quantities of Practical Interest

The dimensionless bulk fluid temperature and the peripherally averaged wall temperature, the local and average Nusselt number at any tube cross section are computed as follow.

The dimensionless bulk fluid temperature is determined as

$$\Theta_{av}(Z) = \frac{1}{\alpha\beta}\int_0^{\alpha}\int_0^{\beta} U(X,Y)\Theta(X,Y,Z)dYdX \tag{19a}$$

from which, evaluating the double integral, one can find that

$$\begin{aligned}\Theta_{av}(Z) = \frac{A^*}{\alpha\beta}\sum_{i=1}^{\infty}\sum_{m=1}^{\infty}\tilde{\tilde{\theta}}_{im}(Z)\sum_{k=1,3,\ldots}^{\infty}E_k\Bigg\{\frac{(-1)^{\frac{k-1}{2}+i-1}B_i C_m}{2}\left[\frac{1}{\mu_i + a_k} - \frac{1}{\mu_i - a_k}\right]\times \\ \times\left[\beta\delta_{1m} + (-1)^m \frac{a_k \tanh(a_k\beta)}{a_k^2 + \lambda_m^2}\right] + \frac{(-1)^{k-1}}{a_k^2}\left[\frac{\alpha}{2} - \frac{1}{\alpha\, a_k^2}\right]\left[\beta - \frac{\tanh(a_k\beta)}{a_k}\right]\times \\ \times\left[\frac{\beta^2}{6} + \frac{1}{a_k^2} - \left(\frac{\beta}{2a_k} + \frac{1}{\beta a_k^3}\right)\tanh(a_k\beta)\right]\Bigg\}.\end{aligned} \tag{19b}$$

It can be readily shown by using the First Law of Thermodynamics that the dimensionless bulk fluid temperature, for problems in which all boundary conditions are of the second kind, is given in the following form, remarkably less involving than Eq. (19b).

$$\Theta_{av}(Z) = 4Z \tag{19c}$$

The dimensionless average wall temperature is given by

$$\Theta_{w,av}(Z) = \frac{1}{\alpha+\beta}\left[\int_0^{\alpha}\Theta(X,\beta,Z)dX + \int_0^{\beta}\Theta(\alpha,Y,Z)dY\right] \tag{20a}$$

which, after evaluating the integrals, yields

$$\Theta_{w,av}(Z) = \frac{1}{\alpha+\beta}\left\{\sum_{i=1}^{\infty}\sum_{m=1}^{\infty}B_i C_m \tilde{\tilde{\theta}}_{im}\left[(-1)^{m-1}\alpha\delta_{i1} + (-1)^{i-1}\beta\delta_{1m}\right] + \alpha\beta + \frac{\alpha^2+\beta^2}{6}\right\}. \tag{20b}$$

The Nusselt number can be evaluated from the balance between the heat flux input and the heat convection output at the wall, and can be written as

$$Nu_1(Z) = \frac{h(Z)D_h}{k} = \frac{1}{\Theta_{w,av}(Z) - \Theta_{av}(Z)}. \tag{21a}$$

An alternative way to compute the Nusselt number arises from the use of Eq. (19c), yielding

$$Nu_2(Z) = \frac{h(Z)D_h}{k} = \frac{\frac{d}{dZ}\Theta_{av}(Z)}{4\left[\Theta_{w,av}(Z) - \Theta_{av}(Z)\right]}. \tag{21b}$$

so that for the fully converged solution, Eqs. (21a) and (21b) should yield the same numerical result, providing an interesting check of the convergence behavior.

The average Nusselt numbers are then computed from

$$Nu_{1,av}(Z) = \frac{h_{1,av}(Z)D_h}{k} = \frac{1}{Z}\int_0^Z Nu_1(Z)dZ\,; \tag{22a}$$

$$Nu_{2,av}(Z) = \frac{h_{2,av}(Z)D_h}{k} = \frac{1}{Z}\int_0^Z Nu_2(Z)dZ\,. \tag{22b}$$

Unless explicitly stated, the local and average Nusselt number that appear bellow refers to Nu_1 and $Nu_{1,av}$, computed using Eq. (21a) and (22a), respectively.

4. RESULTS AND DISCUSSION

System (17), truncated with $N \leq 20$ terms, has been solved using the DIVPAG subroutine from the IMSL package to illustrate the convergence behavior of the present approach, with Z ranging from 10^{-4} to 10^{2}. Figure 2 shows a convergence comparison of $Nu_1(Z)$ and $Nu_2(Z)$, as obtained from Eqs. (21a) and (21b), for a square duct ($\alpha^* = b/a = 1$). For all the tested values of N, the results from these two expressions are practically coincident throughout the considered Z domain. As one can see, for values of Z greater then 3×10^{-3}, the calculated Nusselt numbers lie over a single curve, reaching a fully developed value of 3.087 when Z is bigger than unity. Besides, increasing the value of N towards infinity, the solution of the truncated system (17) approaches the exact one, given by Eqs. (11). For practical purposes, as shown in Fig. 2, a good approximation is obtained for $N = 15$.

Figure 3 correspond to the dimensionless bulk and average wall temperature along the thermal entry region of rectangular ducts for different aspect ratios. It can be noted that the bulk fluid temperature curves lie all together for every aspect ratio, and agree completely with Eq. (19c). It is noticeable that the average wall temperature increases non-linearly from the inlet until about Z = 0.1. Afterwards, it begins to behave linearly-like, tending to become parallel to the bulk fluid temperature curve, as the axial coordinate further increases.

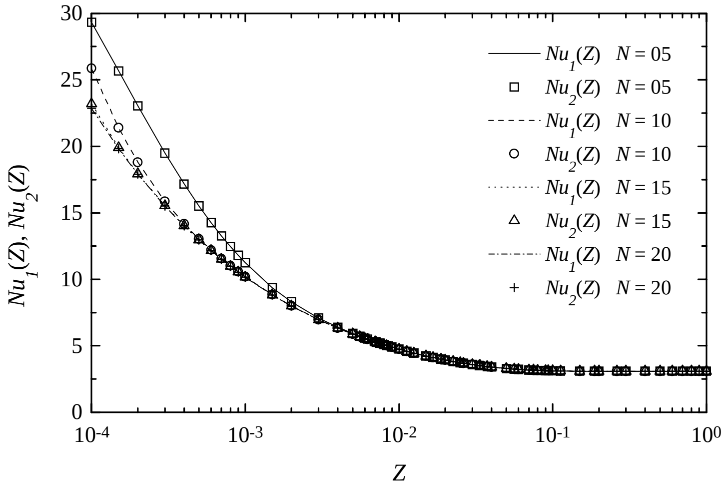

Figure 2 - Convergence of local Nusselt number for a square duct ($\alpha^* = b/a = 1$).

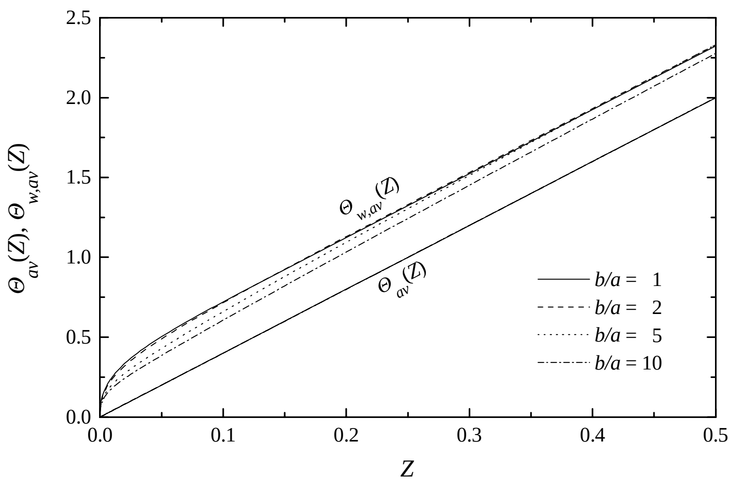

Figure 3 - Dimensionless temperature profiles for rectangular ducts for several aspect ratios.

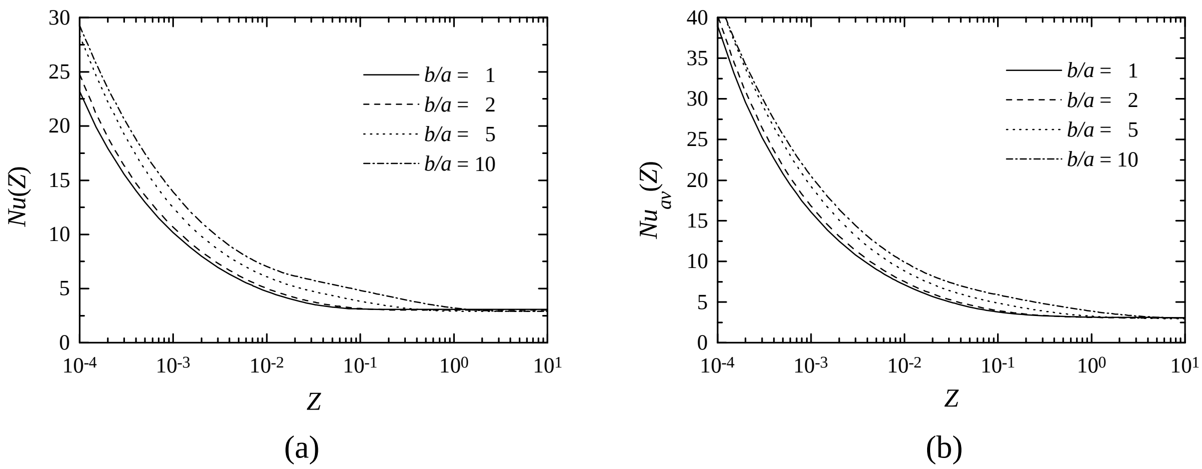

Figure 4 – Local (a) and average (b) Nusselt number in the thermal entry region of rectangular ducts for different aspect ratios.

Figure (4a) and (4b) presents, respectively, the local and average Nusselt number in the thermal entry region of rectangular ducts for several aspect ratios. As one can see, the curves are relatively separated near the inlet region, becoming closer as the axial coordinate approaches unity, from where they assume their fully developed value.

Table 1 presents a comparison between the present results and the available in the open literature, employing a discrete least squares method, showing good agreement.

Table 1 – Comparison of results for fully developed Nusselt number.

Aspect Ratio	Fully Developed Nusselt Number	
($\alpha^* = b/a$)	Present work	Shah & London (1978)
1	3.09	3.09
2	3.02	3.02
5	2.92	2.93
10	2.91	2.95

Acknowledgments

The first author wishes to acknowledge the financial support provided by FAPESP, *Fundação de Amparo à Pesquisa do Estado de São Paulo* (grant No. 97/12818-3).

REFERENCES

Aparecido, J. B. and Cotta, R. M., 1990a, Analytical solutions to parabolic multi-dimensional diffusion problems within irregularly shaped domains, In: Wrobel, L. C., Brebia, C. A., Nowac, A. J., (Eds.), Advanced computational methods in heat transfer, Springer-Verlag, Berlin, vol.1, pp.27-38.

Aparecido, J. B. and Cotta, R. M., 1990b, Thermally developing laminar flow inside rectangular ducts, International Journal of Heat and Mass Transfer, vol.33, n.2, pp.341-347.

Aparecido, J. B. and Cotta, R. M., 1992, Laminar thermally developing flow inside right-angularly triangular ducts, Applied Scientific Research, n.49, pp.355-368.

Aparecido, J. B., 1997, How to choose eigenvalue problems when using generalized integral transforms to solve thermal convection-diffusion problems, In: Proceedings of 14th Brazilian Congress of Mechanical Engineering, Bauru, SP, Brasil.

Cotta, R. M., 1993, Integral transforms in heat transfer and fluid flow, CRC Press, Boca Raton.

Özisik, M. N. and Murray, R. M., 1974, On the solution of linear diffusion problems with variable boundary condition parameters, ASME Paper No.74-HT-1.

Mikhailov, M. D. and Ozisik, M. N., 1984, Unified analysis and solutions of heat and mass diffusion, John Wiley, New York.

Shah, R. K. and London, A. L., 1978, Laminar flow forced convection in ducts, In: Heat Transfer Suppl.1, Academic Press, New York.

Visual Numerics, 1994, IMSL Math/Library, Edition 10, Version 2.0, Suite 400, 9990 Richmond Avenue, Houston, TX-77042.